

## LATTICE SUPERSYMMETRY AND STRING PHENOMENOLOGY

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I discuss the usefulness of lattice supersymmetry in relation to string phenomenology. I suggest how lattice results might be incorporated into string phenomenology. I outline difficulties and describe some constructions that contain an exact lattice version of supersymmetry, thereby reducing fine-tuning of the regulator. I mention some problems that occur for these lattices.

*String phenomenology would benefit from lattice supersymmetry.* Without stabilization of moduli, very little predictivity exists in semi-realistic (supersymmetric) string-derived models. The details of stabilization schemes are always somewhat “iffy” because of nonholomorphic quantities in the effective theory (things that depend on the Kähler potential  $K$ , such as soft masses, B-terms, A-terms, and more generally, the scalar potential), which are not protected by nonrenormalization theorems. Without such theorems, one can only (reliably) resort to: (i) symmetry constraints on the effective theory; (ii) calculations in a weak coupling regime; (iii) direct study of nonperturbative physics. Typically, (i) and (ii) are not powerful enough to yield the reliability that people like myself long for. On the other hand, (iii) may allow us to explore some important issues in string phenomenology.

Naturally, dynamical supersymmetry breaking through the physics of a strongly coupled gauge theory is a realm in which we would like to improve our understanding of nonperturbative physics; in particular, nonperturbative renormalization of the Kähler potential.

But there are other aspects of the low energy effective theory that depend on our understanding of strongly coupled gauge theories. For example, it is well known that various *ad hoc* assumptions for nonperturbative corrections to the Kähler potential of the dilaton can stabilize it at a weak coupling value;<sup>1</sup> what is not as well appreciated is that *generic* nonperturbative corrections to the Kähler potential of the dilaton carve out local minima in the strong coupling regime.<sup>2</sup> Until we understand the Kähler potential in the strong coupling regime, we cannot really say where the global minimum is. Even if we fine-tune beta function coefficients to yield a viable race-track stabilization,

a deeper minimum may exist at strong coupling once the nonperturbative super-Yang-Mills (SYM) corrections to the Kähler potential are included.

The dilaton is just one of many moduli  $\phi_i$  whose stabilization we ultimately need to understand. Thus a more general statement of the observations made above is the following. *Very often in semi-realistic string-derived models there exist points in moduli space where at least one factor in the gauge group becomes strongly coupled in the infrared. In such cases we often cannot reliably say what the true vacuum of the theory is without an improved understanding of the strongly coupled gauge theory.* How can we say what are the deepest minima of the scalar potential  $V(K, W; \phi_i, \bar{\phi}_i)$  if we only know  $K$  in some subspace of the scalar manifold? Lattice supersymmetry might be able to yield at least qualitative results related to this issue.

*Nonperturbative super-QCD and supergravity.* Consider how we typically incorporate super-QCD instantons into an effective supergravity theory. We obtain an effective action  $S_{\text{eff}}(g, \theta, \langle \phi_i \rangle)$  that describes the instanton corrections from super-QCD, which (unlike the supergravity we are supposed to be studying) is a renormalizable field theory; the  $\langle \phi_i \rangle$  denotes that we often fix some scalars to some point in the moduli space; we also work at a fixed value of the gauge coupling  $g$ , and the  $\theta$  parameter dependence is generally implied by holomorphy. Then we embed the instanton corrections back into the supergravity effective action. We argue that it is okay to compute the instanton corrections from super-QCD rather than the full supergravity because the effects of the nonrenormalizable supergravity operators in the effective theory will be suppressed by the dynamical/fundamental hierarchy  $\Lambda_{\text{SQCD}} \ll m_P$ . There is often a self-consistency criterion implicit in all of this; e.g., since  $\Lambda_{\text{SQCD}}$  depends on the dilaton, taking  $g, \theta$  to be frozen background fields in the instanton calculation assumes that the dilaton is sufficiently stabilized that we can treat it this way; otherwise the results of the computation with the dilaton held frozen may have little to do with the actual instanton corrections in the theory with a dynamical dilaton (particularly where the Kähler potential is concerned).

By analogy, I would like to make the following “lattice proposal” for studying nonperturbative corrections to string-derived effective supergravity actions: obtain  $S_{\text{eff}}(g, \theta, \langle \phi_i \rangle)$  from lattice super-QCD. Provided it is asymptotically free, the *target continuum theory* has an ultraviolet attractive fixed point, and therefore a corresponding lattice theory should have a well-defined continuum limit. The effective action will be obtained by the usual procedure of matching lattice results for Euclidean Green’s functions to predictions of an effective (interpolating) continuum action, up to errors of order  $a\Lambda_{\text{SQCD}}$ ,

where  $a$  is the lattice spacing. We then embed these results into the “tree” supergravity effective action, just as we do the instanton calculations. In this way, the lack of *universality* associated with a nonrenormalizable theory such as supergravity is no big deal; it is just the usual regulator sensitivity that one faces when computing quantum corrections to an effective field theory. What is important is that our regulator respect the symmetries of the target continuum theory. The lattice action neglects nonrenormalizable supergravity interactions; thus we expect to obtain results that are valid up to order  $1/amp$  corrections, as well discretization and finite volume systematic errors. What we need is a “window” where  $m_P \gg a^{-1} \gg \Lambda_{\text{SQCD}}$ . In this way the lattice computation avoids the physics of the underlying theory near the Planck scale. Note that we will not be able to explore regions of moduli space where this inequality fails to hold; for this we need a nonperturbative analysis of string theory itself. We must also check the self-consistency of holding moduli, such as the dilaton, fixed, just as we did in the case of the instanton calculation.

*Lattice supersymmetry is problematic.* At most a discrete subgroup of the super-Poincaré invariance group can be realized exactly on the lattice, since it already breaks the Poincaré invariance to a subgroup. Moreover, it is well-known that chiral symmetries are difficult to realize on the lattice;<sup>3</sup> chiral R symmetries and flavor symmetries play a key role in super-QCD.

“It’s just a regulator,” one might say. True enough, in principle a combination of (i) adding counterterm operators to the action, and (ii) fine-tuning bare lattice parameters, can yield the correct *quantum continuum limit*: the continuum limit of lattice Green’s functions involving energy-momentum scales well below the cutoff should satisfy the various Ward identities. However in practice fine-tuning and a complicated lattice action are “expensive;” the computations take too long since the Green’s functions must be evaluated by Monte Carlo simulation over a range of bare parameters in order to find a point that will satisfy the Ward identities. This can render the fine-tuning approach impractical. Nevertheless quite a bit of work in this direction has been done; e.g., see refs. 4.

*Exact lattice symmetries.* For the hypercubic lattice actions that are most often studied, discrete rotation and translation symmetries guarantee Poincaré invariance in the quantum continuum limit, without any fine-tuning: the exact lattice symmetries prevent one from writing down relevant or marginal operators that would violate Poincaré invariance in the *interpolating continuum action* (the effective action that describes the quantum continuum limit of the lattice theory). Long ago, Ginsparg and Wilson pointed out that there can exist an exact lattice version of chiral symmetry that

will likewise guarantee the continuum chiral symmetry without fine-tuning.<sup>5</sup> There now exist realizations of the Ginsparg-Wilson relation that the lattice Dirac operator must obey, introduced in the quite famous papers of Kaplan, Neuberger<sup>6</sup> and Lüscher.<sup>7</sup> In the latter paper Lüscher suggested the possibility of an exact lattice supersymmetry. This notion has been explicitly realized by several groups; for example, see refs. 8-11 and references therein. In the examples that are interacting, the supercharges are nilpotent,  $Q^2 = 0$ , allowing for an algebra that is consistent with the absence of infinitesimal Poincaré invariance on the lattice. The exact lattice supersymmetry forbids certain relevant or marginal operators. This eliminates or reduces the fine-tuning required to obtain the target continuum theory. One rather interesting idea is based on deconstruction; it is due to Cohen, Kaplan, Katz and Ünsal.<sup>8-10</sup> They are able to write down lattice Yang-Mills theories with some exact lattice supersymmetry. Brief reviews by Kaplan are available; these are readable and supply details that I do not have space to provide.<sup>12</sup>

*Deconstruction, originally.* This is a gauge-invariant regularization of  $d > 4$  gauge theories. One obtains an effective  $d > 4$  gauge theory from a 4d *quiver = moose = product group gauge theory*.<sup>13</sup> 1 or 2 dims are *emergent*, effective dimensions. In Fig. 1, I show a quiver theory with 6 factors  $G_1, \dots, G_6$  and representations  $(\mathbf{R}_i, \mathbf{R}_j)$  that are charged under pairs, forming “links” of a “torus” in “theory space.”

*Deconstruction, latticization.* Fig. 1 cries out: Why not make some/all of our 4d emergent? Is this a pathway to new lattice theories? Maybe if the quiver is supersymmetric, will we wind up with some exact lattice supersymmetry?

Deconstruction of all 3 spatial dimensions was considered by Kaplan, Katz and Ünsal.<sup>8</sup> Deconstruction of just 1 spatial dimension was considered by Poppitz, Rozali and myself;<sup>14</sup> we looked at the  $U(1)_R$  anomaly in  $\mathcal{N} = 2$  4d SYM. Poppitz has gone on to analyze KK monopoles in this framework.<sup>15</sup>

The case with all dimensions latticized has also begun to be studied. Here one starts with 0d supersymmetric quiver theories. This affords a spacetime (Euclidean) lattice formulation of SYM, and explicit constructions have been written down by Cohen et al.<sup>9,10</sup> These 0d theories contain some exact lattice supersymmetry that eliminates or reduces fine-tuning. I have studied some of the practical aspects of these theories;<sup>16,17</sup> i.e., relating to actually running a Monte Carlo simulation that would compute Euclidean Green’s functions. I will now outline a couple of the simpler models and comment very briefly on my recent findings.

(2,2) 2d  $U(k)$  SYM. This model was introduced in ref. 9. The target theory is (2,2) 2d  $U(k)$  SYM, which is obtained<sup>a</sup> from a 4d  $\rightarrow$  2d reduction of  $\mathcal{N} = 1$  4d pure SYM. We start with a *mother theory*. It is the  $\mathcal{N} = 4$  SYM matrix model that is obtained from 4d  $\rightarrow$  0d reduction of 4d  $\mathcal{N} = 1$   $U(kN^2)$  SYM. The theory contains (Hermitian) bosons  $v_m = v_m^\beta T^\beta$ ,  $m \in \{0, 1, 2, 3\}$  in a Hermitian basis  $T^\beta$  of  $u(kN^2)$ . In addition the theory contains 2-component Euclidean Majorana-Nicolai fermions  $\psi = \psi^\beta T^\beta$ ,  $\bar{\psi} = \bar{\psi}^\beta T^\beta$ ,  $\psi = (\lambda, \xi)^T$ ,  $\bar{\psi} = (\alpha, \beta)$ . The mother theory action is

$$S = \frac{1}{4g^2} \text{Tr} ([v_m, v_n][v_m, v_n]) + \frac{1}{g^2} \text{Tr} (\bar{\psi} \bar{\sigma}_m [v_m, \psi]), \quad \bar{\sigma}_m = (1, i\sigma) \quad (1)$$

Next we progress to a *daughter theory*. We project out all fields *not* inert with respect to a  $Z_N \times Z_N$  symmetry of mother theory; i.e., we orbifold the matrix model. I suppress most of the details. The orbifold is chosen in such a way that it breaks the mother theory gauge group to a  $U(k)$  lattice gauge symmetry:  $U(kN^2) \rightarrow \bigotimes_{\mathbf{m}=(1,1)}^{(N,N)} U(k)_{\mathbf{m}}$ . The bosons which survive decompose as follows:  $v_m \rightarrow x_{\mathbf{m}}, \bar{x}_{\mathbf{m}}, y_{\mathbf{m}}, \bar{y}_{\mathbf{m}}$ , with each a bifundamental linking two factors in the quiver, just as in the 1d quiver mentioned above.  $x$ 's are links in the  $x$ -direction and  $y$ 's are links in the  $y$ -direction.

Finally we extract the *lattice theory*. To do this we expand around a point in moduli space:  $x_{\mathbf{m}} = (2a^2)^{-1/2} \mathbf{1} + \dots$ ,  $y_{\mathbf{m}} = (2a^2)^{-1/2} \mathbf{1} + \dots$ . This is stabilized with deformation of the action:

$$\Delta S = \frac{a^2 \mu^2}{g^2} \sum_{\mathbf{m}} \text{Tr} \left[ \left( x_{\mathbf{m}} x_{\mathbf{m}}^\dagger - \frac{1}{2a^2} \right)^2 + \left( y_{\mathbf{m}} y_{\mathbf{m}}^\dagger - \frac{1}{2a^2} \right)^2 \right] \quad (2)$$

Although this breaks the lattice supersymmetry, we scale the deformation away in the thermodynamic limit:  $\mu \sim 1/Na$ . Further conditions, required to control fluctuations, are described in ref. 9.

*Fermion determinant.* In the daughter theory, the fermion action is of the form  $S_F = \bar{\psi} M \psi$ , and the (boson field dependent) matrix  $M$  is problematic.<sup>16</sup> First, there exists an ever-present fermion zeromode,  $\det M \equiv 0$ , independent of the boson configuration. I project it out to exhibit the determinant for the other fermions. A simple method is to deform the fermion matrix  $M \rightarrow M_\epsilon \equiv M + \epsilon \mathbf{1}$ . Then the product of nonzero eigenvalues is identical to  $\det \hat{M}_0 \equiv \lim_{\epsilon \rightarrow 0^+} \epsilon^{-1} \det M_\epsilon$ . In the computation of Green's functions, this procedure factors out and cancels the zeromode. I have studied  $\det \hat{M}_0$  for boson configurations drawn randomly from a Gaussian distribution, centered on the relevant point in moduli space, with unit variance. The phase

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<sup>a</sup>All SYM theories discussed in the following are the Euclidean versions.

$\phi = \arg \det \hat{M}_0$  is uniformly distributed throughout the interval  $(-\pi, \pi]$  (see Fig. 2).

After integrating out the fermions, we are left with a complex “effective” boson action

$$\exp(-S_{\text{eff}}(x, y)) = e^{i\phi(x, y)} \left[ \exp \left( -S_B(x, y) + \ln |\det \hat{M}(0; x, y)| \right) \right] \quad (3)$$

Presumably the corresponding violation of unitarity vanishes in the continuum limit; certainly we see no evidence of it in the classical continuum limit, which has been worked out in ref. 9. The quantity in brackets is a valid probability measure for boson configurations. The corresponding action is called the *phase-quenched* action:  $S_{p.q.}(x, y) = S_B(x, y) - \ln |\det \hat{M}(0; x, y)|$ . It is difficult to study the original action by Monte Carlo methods, because we must reweight the phase-quenched expectation values by the phase using the identity  $\langle \mathcal{O} \rangle = \langle e^{i\phi} \mathcal{O} \rangle_{p.q.} / \langle e^{i\phi} \rangle_{p.q.}$ . The cancellations that occur as the phase flops around are in most cases a severe problem. The same problem is encountered in QCD with an appreciable baryon density.

(4,4) 2d  $U(k)$  SYM. This is another model constructed by Cohen et al.<sup>10</sup> The target theory is (4,4) 2d  $U(k)$  SYM, obtained from the 6d  $\rightarrow$  2d reduction of  $\mathcal{N} = 1$  6d pure  $U(k)$  SYM. For this model I find similar problems fermion determinant.<sup>17</sup> With random boson pulls from a Gaussian distribution I obtain a roughly flat distribution for the phase of the fermion determinant; the data is quite similar to Fig. 2. One might hope that in the phase-quenched ensemble the situation would improve—that the phase of the fermion determinant would most often be found near some particular value. Unfortunately, I find that this is not the case for the smaller lattices that I can presently simulate; again, the data is flat like Fig. 2.

*Conclusions.* While it would be very nice indeed to obtain reliable and accurate results for supersymmetric Yang-Mills using lattice simulations, it is not at all an easy task. Further research is needed; it remains to be seen whether or not results useful to string phenomenology will emerge.

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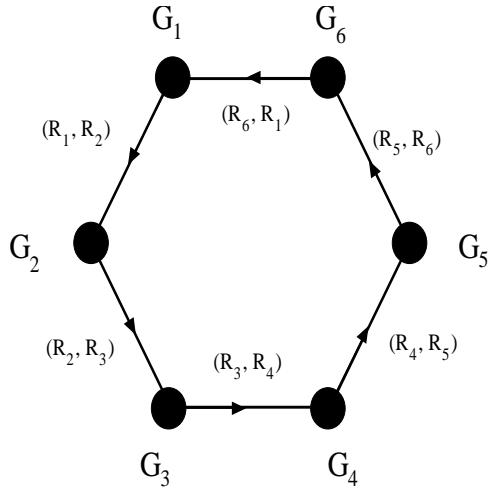


Figure 1. Obligatory quiver diagram.

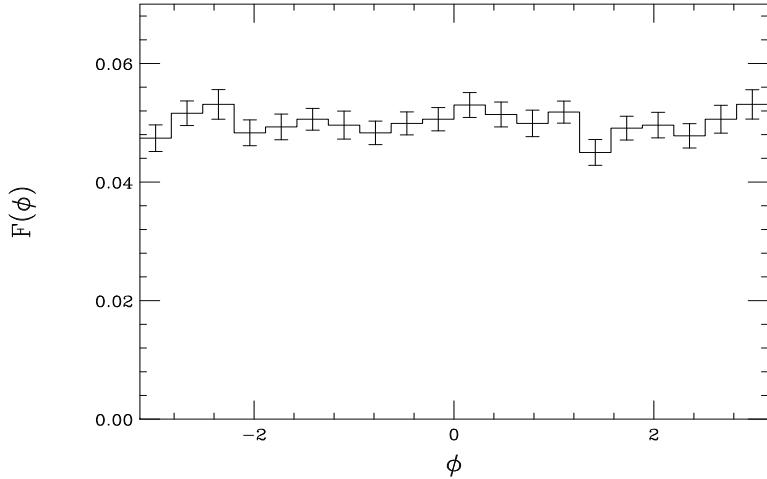


Figure 2. Frequency  $F(\phi)$  of the phase  $\phi = \arg \det \hat{M}_0$  in all bins for a Gaussian boson distribution on a  $6 \times 6$  lattice whose target is (2,2) 2d  $U(2)$  SYM. Smaller lattices give similar results.